Free Spectral Range (FSR) of a Scanning Fabry-Perot Interferometer

A Fabry-Perot cavity (or interferometer, or etalon) is an optical system that has two partially transmitting mirrors facing each other, between which light coming in through one mirror is reflected multiple times before it exits the opposite mirror. These cavities may be designed in various ways. The Model 240 Spectrum Analyzer is a confocal scanning Fabry-Perot interferometer. It consists of two concave partially-reflecting mirrors separated by their radius of curvature, one of which is mounted on a piezoelectric material. (For some reason, the system is drawn somewhat differently in the brochure by Coherent, Inc.) Light entering through one mirror makes four passes through the cavity before exiting. Where it exits in phase, it produces an interference maximum, and where is it out of phase, there is a minimum. The result is a circular fringe pattern. If the light entering the cavity is a beam much narrower than the diameter of the first off-axis fringe, then it will not survive the trip through the cavity unless it exits in phase with itself. For the geometry of the spectrum analyzer in this experiment, this happens when $n\lambda = 4d$, where d is the distance between the mirrors.

Expressed in terms of frequency, $\nu_n = c/\lambda = n(c/4d)$, and the frequencies of light transmitted by the interferometer form a series of interference maxima having a uniform spacing of $\Delta\nu = \nu_n - \nu_{n\text{-}1} = c/4d$. The distance between the mirrors in the model 240 Spectrum Analyzer is about 1.0 cm. This gives $\Delta\nu = (2.998 \times 10^{10}\ \text{cm/s})/(4 \times 1\ \text{cm}) = 7.50 \times 10^9$ Hz, or 7.5 GHz, the *free spectral range* of the particular type of model 240 used in the laser properties experiment. We can call this quantity $\Delta\nu_{\text{FSR}}$.

To vary the wavelength of light transmitted through the interferometer, you apply a ramp voltage, by means of the Model 251 controller, to the piezoelectric mount that holds the movable mirror. The resonance condition above tells us that in order to scan one free spectral range, we need move the mirror by only about one-quarter the wavelength of the incident light. Say that we start with light of wavelength λ, which satisfies $n\lambda = 4d$. Changing the mirror separation by $\lambda/4$ makes the new resonance condition $4(d + \lambda/4) = n\lambda'$, where the wavelength $\lambda' = \lambda + \Delta\lambda$. Substituting gives $4(d + \lambda/4) = n(\lambda + \Delta\lambda)$, from which we find that $\Delta\lambda = \lambda/n$. $n = 4d/\lambda$ gives $\Delta\lambda/\lambda^2 = 1/4d$. From $\nu = c/\lambda$, we have $\Delta\nu \approx c(\Delta\lambda/\lambda^2) = c/4d = \Delta\nu_{\text{FSR}}$, the free spectral range. If we scan further than this, we reach a point where the resonance condition is satisfied again, and we obtain another copy of the spectrum, one free spectral range – 7.5 GHz for our instrument – from the first.

It may seem odd that we can scan through the free spectral range of the interferometer (or further) without changing it significantly. We can calculate the shift in $\Delta\nu_{\text{FSR}}$ caused by a quarter-wavelength shift of the mirror as $\delta(\Delta\nu_{\text{FSR}}) = c/4d - c/(4d + \lambda) \approx (c/4d)(\lambda/4d)$, which gives a relative change of $\delta(\Delta\nu_{\text{FSR}})/\Delta\nu_{\text{FSR}} \approx \lambda/4d$. For the wavelength of the He-Ne laser, 632.8 nm, this equals 632.8 nm/(4 × 1 cm) = 1.6×10^{-5}, a negligible change.

In writing this note, I leaned heavily on a very good explanation from the Columbia University E.K.A. Advanced Physics Laboratory Course, which is available at the following link:

http://www.phys.columbia.edu/~w3081/exp_files/laser_exp.pdf